Vector Hysteresis Modeling Based on Energy Homogenization

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In various applications, the magnetic hysteresis modeling must ensure that both the magnitude and direction of the magnetization are quantified correctly. To this end, extensive studies have been made in the context of Preisach modeling. Inspired by the recently proposed Preisach-Stoner-Wohlfarth model, a magnetic hysteron, is proposed in a more generalized energy framework, whose behavior is in agreement with the required thermodynamic properties. Using the information conducted by this hysteron, an engineering based simplification of the characteristics of the hysteron and a systematic treatment of its distribution on a quasi-Preisach diagram can be established. In this paper, an efficient and self-consistent vector hysteresis model is proposed. The proposed model simulates the macroscopic ferromagnetic properties in the real media and is validated experimentally.

***Index Terms*—Critical surface, micromagnetics, Preisach modeling, vector hysteresis.**

I. Introduction

THE MAGNETIZATION behavior of ferromagnetic materials is known to be of vectorial characteristics. However, the classical Preisach model can only be applicable to hysteresis subjected to an alternative external magnetic field. This drawback can be addressed by various modifications.

Straightforward generalization of the classical Preisach modeling proved to be either incomplete or full of artificial assumptions, which results in unphysical characteristics and inefficient parameter identifications. Recently, a novel hysteresis modeling is constructed. Depending on the critical surfaces of the proposed hysteron, the amplitude variation of magnetization is determined [1]. To address the angular variation of magnetization, the famous Stoner-Wohlfarth (SW) model is employed [2]. The combination of these two elements creates an axis independent model, requiring no assumptions on the rotational behavior of the magnetization dynamics. In addition, the complicated computation associated with the SW model can be evaluated once-for-all. However, the introduction of the critical surface and the employment of SW model are inherently inconsistent. In addition, it is not easy to find the identification parameters used in the critical surface and SW model. In this paper, all these problems are circumvented by the deduction of the hysteron based on the theory of phase transition.

II. Micromagnetics Based Hysteron

A. Energy of Ferromagnetic Material

The ferromagnetic system being considered is an array of fixed sites forming a periodic lattice. A given set of spins $\{\boldsymbol{S}_i\}$ specifies a configuration of the whole system. The energy in this configuration takes the form of

$$E = -\frac{1}{2} J \sum_{i \neq j} \boldsymbol{S}_i \cdot \boldsymbol{S}_j - \mu_0 \boldsymbol{H} \cdot \boldsymbol{M} \tag{1}$$

The first term is referred as the exchange energy, by which the electrons spinning in the magnetic materials are lined up in parallel. J denotes the strength of interaction among neighbouring spins. The second terms represents the Zeeman energy due to the external magnetic intensity $\boldsymbol{H}$.

B. Formulation of Hysteron

Because the evolution of the rate-independent hysteresis can be regarded as a sequence of states of equilibrium, the magnetization can be determined by the equilibrium state of the energy represented by (1). Assuming that there are sufficient degenerative energy levels in the ferromagnetic materials so that the spins $\{\boldsymbol{S}_i\}$ are classical vectors, the magnetic dipole moment $\boldsymbol{M}$ in that system for a particular spin configuration is given by

$$\boldsymbol{M} = \sum_{i}^{N} \boldsymbol{S}_i \tag{2}$$

To apply statistical mechanics to such subsystem, the partition function Z must be firstly determined in order to evaluate other macroscopic thermodynamic quantities. For classical spins, the canonical distribution is readily applied. However, the partition function of the system involves integration, rather than summation, over these variables. In two-dimensional spin vector space, equal probabilities with equal solid angle $d\Omega$ change the partition function into

$$Z = \int \frac{1}{(2\pi)^N} \exp(-\beta E) \prod_{i=1}^{N} d\Omega_i \tag{3}$$

To facilitate the above integration, mean-field approach is utilized, i.e., assume the exchange energy can be approximated as

$$-\frac{1}{2} J \sum_i \boldsymbol{S}_i \cdot \sum_{<j>i} \boldsymbol{S}_i = -\frac{1}{2} J \sum_i \boldsymbol{S}_i \cdot \gamma \boldsymbol{m} \tag{4}$$

i.e., the sum over pairs can be split into a sum over all the spins and the nearest neighbours of spin i, $<j>i$. The latter is further approximated to be proportional to the normalized magnetization $\boldsymbol{m}$ with respect to the saturated magnetization $\boldsymbol{M}_s$. The total energy of the system can thus be rewritten as

$$E = -\mu_0 \left(\frac{J\gamma}{2\mu_0} \boldsymbol{m} + \boldsymbol{H}\right) \cdot \boldsymbol{M} = \mu_0 H_{eff} \cdot \boldsymbol{M} \tag{5}$$

Inserting (5), the integration of (3) gives

$$Z = [I_0(\mu_0 \beta H_{eff})]^N \tag{6}$$

where I_0 is the zeroth order modified Bessel function of the first kind. $\beta = 1/k_B T$, and k_B is the Boltzmann constant. The evaluation of average $\boldsymbol{m}$ is known to be ($\langle\cdot\rangle$ denotes expectation)

$$\langle \boldsymbol{m} \rangle = -\frac{1}{\beta}\frac{\partial \ln(Z)}{\partial \mu_0 \boldsymbol{H}_{eff}} = \boldsymbol{n}\, I_1(\mu_0 \beta H_{eff}) / I_0(\mu_0 \beta H_{eff}) \tag{7}$$

where N is the number of spins in the lattice and $\boldsymbol{n}$ represents the unit direction vector of $\boldsymbol{H}_{eff}$. For scalar cases, the procedure of (3) to (7) can be used to obtain the result, $M = \tanh(\mu_0 \beta H_{eff})$, which is identical to those obtained using Ising's model. It is noted that the individual atomic magnetic moments within a domain are aligned in the same direction. Their summation offers a kind of super-magneton, which is equivalent to a significant increase of β in (6).

III. Model Implementation

Given the Preisach distribution of hysterons, $\mu(\Omega)$, the assembly of the hysterons $\gamma(\Omega)$ described in Section of II.B gives the magnetization $\boldsymbol{M}$ induced by the applied field $\boldsymbol{H}$

$$\boldsymbol{M} = \int_{\Omega} \mu(\Omega)\gamma(\Omega) d\Omega \tag{8}$$

where Ω denotes the parameter set to depict the magnetic state of a hysteron completely. According to (7), two significant properties can be drawn:

i) The operator $\gamma(\Omega)$ provides a sigmoid-shaped hysteresis loop on a certain direction. For the two dimensional vector hysteresis, every direction should have the same probability to induce the magnetization since the magnetization in (7) depends on the radial vector. The envelope of these hysterons forms a circular ring critical surface in the H space. the radius and half-width of each critical ring, i.e., H_i and H_c, denote the components comprise H_{eff} other than H and pinning field, respectively. The relationship between H, H_i and the direction vector of magnetization, m, is illustrated in Fig. 1.

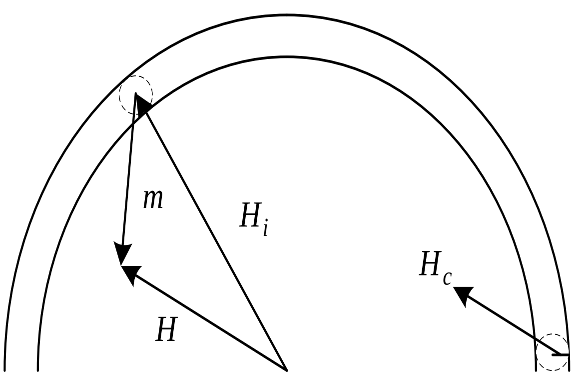

Fig. 1. critical ring of the hysterons in H space

ii) Using H_i and H_c described in above subsection, it is possible to construct a quasi-Preisach diagram in each direction, as shown in Fig. 2, which indicates its compatibility with the classical scalar Preisach modeling. If the external field H rotates, taking Fig. 2 for instance, the magnetization signified within the triangle within pattern 1 will follow H instantaneously. The other parts, both the rectangular created by the previous reversal field H_1 and the initial magnetization state without pattern are frozen in the former direction.

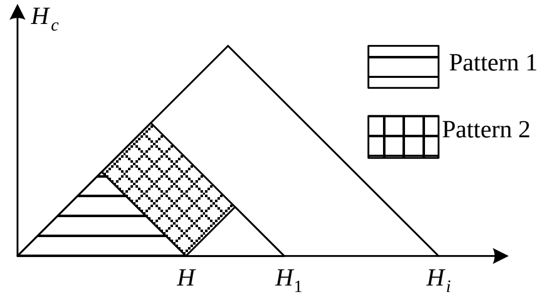

Fig. 2. quasi-Preisach diagram

A. Stack of Magnetization History

A stack is used to efficiently find the previous change of external field, including both magnitude and direction. Different from the stack employed in traditional Preisach modeling, this stack consists of three different reversals, the local minimum field point, the local maximum field point, and the rotated field point.

B. Preisach Distribution

In the proposed model, the Preisach distribution function is defined as the number of hysterons associated with each critical ring. Given maximum H_i and H_c, it can be discretized into a group of concentric rings with a resolution d. Assume the density p_j in the jth ring is uniform, the input sequence, $[\boldsymbol{H}_k]_{k=1,\ldots,m}$, is then corresponding to $[\boldsymbol{M}_k]_{k=1,\ldots,m}$. By writing this mapping in the form of matrix, i.e.,

$$[\boldsymbol{M}_k] = [\gamma(\boldsymbol{H}_k, H_c^j, H_i^j)][p_j] \tag{9}$$

where $k = 1,\ldots m$, $j = 1,\ldots,n$, it is clear that the density function $[p_j]$ can be easily identified by means of least square method or optimization routines.

IV. Numerical Results

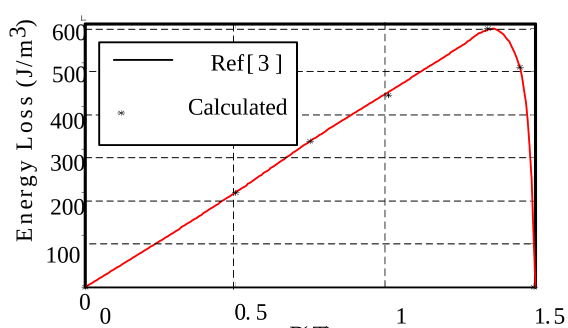

Fig. 3. Rotational loss varying with the magnitude of the rotating flux density

By using the major loop illustrated in [3], the same rotational hysteresis loss is predicted.

V. References

[1] E. D. Torre, E. Pinzaglia and E. Cardelli, “Vector modeling - Part I: Generalized hysteresis model,” *Physica B: Condensed Matter*, vol. 372, no. 1-2, pp. 111-114, 2006.

[2] E. Cardelli, E. D. Torre and E. Pinzaglia, “Numerical Implementation of the Radial Vector Hysteresis Model,” *IEEE Trans. Magn.*, vol. 42, no. 4, pp. 527-530, 2006.

[3] E. Dlala, A. Belahcen, K. A. Fonteyn, and M. Belkasim, "Improving loss properties of the mayergoyz vector hysteresis model," *IEEE Trans. Magn.*, vol. 46, no. 3, pp. 918-924, 2010.